

Reviews of Modern Physics Style Guide

XVI. MATHEMATICAL MATERIAL

Mathematical symbols must be defined immediately where they are introduced. Exceptions are the fundamental constants: the velocity of light c , Planck's constant \hbar , the electronic charge e , Boltzmann's constant k or k_B , and the electron mass m_e .

Avoid using the same symbol for two different things. If you think that a list of symbols would be helpful to your readers, you can provide one preceding the references (see Sec. ??).

In what follows, only the style of equations and symbols will be considered. Authors are urged to consult $\LaTeX 2_\epsilon$ and $\text{REV}\TeX 4$ documentation for instructions on using these packages to display mathematics.

A. Characters

1. Character fonts

The italic font is used for mathematical symbols (this is the default font in \LaTeX 's math mode). In addition to variables and constants, the italic font is used for particle symbols, symbols of quantum states, and group-theoretic designations.

In general please use the following hierarchy of font styles for symbols:

TABLE VI Font style for symbols.

Font style	Symbol	Example
lower case	variables, constants, and ordinary functions	x, α, f
upper case	matrices and functions	S, F
script upper case	operators	\mathcal{H}
boldface lower case	three-vectors	\mathbf{r}
boldface upper case	matrices three-vectors	\mathbf{J}, \mathbf{B}

2. Diacritical signs

A *diacritical sign* is a marking placed directly above or below symbols, e.g., the arrow in \vec{x} . It is possible to make multilevel marks – placing several diacritical marks above or below one letter or symbol – but this quickly becomes confusing. Restrict the number of such marks to two to avoid confusion. The underline can appear under any configuration.

For your convenience, Table VII contains a list of commonly used diacritical marks along with the \LaTeX code that produces them.

TABLE VII Diacritical marks and associated \LaTeX codes.

Diacritical mark	\LaTeX code
\vec{x}	$\backslash\text{tensor}\{x\}$
\dot{x}	$\backslash\text{dot}\{x\}$
$\ddot{\mathbf{r}}$	$\backslash\text{ddot}\{\{\backslash\text{bf r}\}\}$
$\hat{\theta}$	$\backslash\text{dot}\{\hat{\{\theta\}}\}$
\underline{A}	$\backslash\text{underline}\{A\}$
$\overline{\overline{x + y}}$	$\backslash\text{overline}\{\overline{x + y}\}$

3. Subscripts and superscripts

All available characters can be used as subscripts or superscripts. Position of subscript or superscript is dictated by standard notation. In almost all cases you should set right and left subscripts and superscripts flush against the symbol they accompany (as in the following).

Examples:

$$R_0^x, {}^{110}\text{Ag}^m, \rho_0^{(N)}, \int_0^1, \sum', \lim_{l \rightarrow \infty} f_{ly} \text{ (in text)}$$

There are, however, some exceptions to this general rule. Examples appear below.

- tensor notation: $g_{\mu\nu}(\phi^z)^{\alpha;\alpha}$
- molecular ions: $\text{H}_2^+, \text{O}_2^-$
- footnotes in tables: E_n^a

Presuperscripts or presubscripts are set flush against the symbols they accompany. In addition, it is advisable to insert an extra thin space between a presuperscript or presubscript and a preceding symbol in cases where clarity is questionable, i.e.,

$$8p\sigma^1\Sigma_u^+ \quad \text{or} \quad d^9s^z p^3P_2$$

The number of levels of subscripts and superscripts attached to a symbol will also affect clarity. Two double levels is generally considered the most complicated combination acceptable, i.e.,

$$M_{b_k^\dagger}^{a_i^2}$$

When additional indices are needed, insert a comma or thin space and keep the added indices on the same line, i.e.,

$$M_{b_k, d_p}, \quad \sigma_{r, s+1}, \quad \text{or} \quad \sigma_{r\ s+1}$$

B. Abbreviations in math

Some abbreviations, such as those for mathematical functions and those used in superscripts or subscripts, require special handling and are discussed below.

1. Abbreviations designating mathematical functions

Multiletter abbreviations of mathematical functions are always written in the roman font (i.e., \sin). The standard trigonometric functions are \cos , \cot , \sec , \sin , and \tan . Hyperbolic trigonometric functions add “h” to the end; the preferred notation for inverse functions is \tan^{-1} rather than \arctan . The preferred notation for the logarithm to the base e (\log_e) is “ln”; “log” without a subscript denotes the logarithm to the base 10 (\log_{10}).

The following guidelines are in general use in clear mathematical writing:

- (a) roman multiletter abbreviations must be closed up to the argument following and separated from any preceding symbol by a thin space, that is,

$$K \cos[Q(z - z_0)]$$

$$K \exp[x^2(b_2 + b_1)^{-\frac{1}{2}}]$$

- (b) In addition, by convention it is assumed that an argument ends as soon as another function appears, i.e., $\sin x \cos b$, or at a plus or minus sign, i.e., $\sin x + y$, but if other mathematics is involved or there is any ambiguity you should insert bracketing, as in the following examples:

$$\sin[-(x + a)], (\sin x)/a, \text{ and } \exp[x^2(b_z + b_1)^{-\frac{1}{2}}]$$

- (c) To treat a function of a function enclose it in bold round parentheses, i.e.,

$$g(f(x))$$

- (d) e and \exp (for exponent) notation follow both of the preceding conventions. Which form to use, e or \exp , is determined by the number of characters and the complexity of the argument. The e form is appropriate when the argument is short and simple, i.e., $e^{i\mathbf{k}\cdot\mathbf{r}}$, whereas \exp should be used if the argument is more complicated.

2. Abbreviations in subscripts and superscripts

Abbreviations in subscripts and superscripts fall into two categories: (1) single letter and (2) multiletter. Most single-letter abbreviations are conventionally printed in the italic font, i.e., E_C where C stands for Coulomb. Multiletter abbreviations are conventionally printed in the roman font whether they represent one or more words,

i.e., E_{lab} , where lab stands for laboratory and E_{HF} , where HF stands for Hartree and Fock, two proper names. Please note that you should always capitalize abbreviations that represent proper names.

When you are creating your own abbreviations, do not put periods in acronyms (whether in line or in subscripts), but do insert them if you are abbreviating words that are headings in a table.

C. Mathematical expressions

1. When to display

Mathematical material that is set apart from the main text in the traditional manner is referred to as *displayed* material. In general, authors should observe the following guidelines: Display (1) equations of importance, (2) all equations that are numbered, (3) those that are too long to fit easily in text (over 25 characters), or (4) those that are complicated (contain built-up fractions, matrices, or matrixlike expressions). Consider, also, displaying math that contains multilevel indices, integral, summation, and product signs, with multilevel or complex limits, or any other situation in a formula that creates the need for extra vertical spacing in a text line.

2. Punctuation

Even though displayed math is separated by space from the running text it still is a part of that text and needs to be punctuated accordingly. The following is an example.

The final result is

$$H_{ij} = \left(\frac{\Omega}{\Delta}\right)^2 \frac{|J|^2}{E_g + \frac{1}{2}(W_c + W_v)} e^{\lambda\mathbf{K}\cdot\mathbf{R}_{ij}}, \quad (1)$$

where

$$\mathbf{K} = \frac{1}{a}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \quad (2)$$

and

$$\lambda = \ln [W_l W_v / (12E_g)^2]. \quad (3)$$

Note the use of the comma at the end of the first equation, and the period at the end of the third equation.

3. Equation breaking (multiline equations)

Mathematical expressions often need to be displayed on two or more lines (“broken”) because of the line length limitations of the *Reviews of Modern Physics* standard

two-column layout. The best place for a break is just before an operator or sign of relation. These signs should begin the next line of the equation. When it is necessary to break a product, begin the continued line with a multiplication sign (\times). Note that the material that comes after a break can and should be aligned so that its relationship to the material on the first line is mathematically correct. Here is an example:

$$\begin{aligned} N_x(\mathbf{r}) + iN_2(\mathbf{r}) &= e^{i\theta(\mathbf{r})} \\ &= \exp[-ij_x u(\mathbf{r})]. \end{aligned} \quad (4)$$

Equations that are not displayed, but appear in text may also need to be broken. Basically the same rules apply as when breaking displayed math. Breaking at an operator or sign of relation is best. The operator or sign of relation usually begins the next line of text:

... their respective displacement vectors $a\hat{\mathbf{i}}/2$
 $+\sqrt{2}a\hat{\mathbf{j}}/2$...

Products are broken with a multiplication sign:

... keep $\delta m = 4$ MeV and choose $\gamma (= -5.46 \times 10^{-3})$ at ...

In addition, you may break in text at a solidus, leaving the numerator and fraction bar on the top line. The denominator will begin the continued line as follows:

... \mathbf{J} is proportional to $T_0(\Delta_1, \Delta_3)^{1/2}/$
 $N_2(0)$...

4. Equation numbering, special situations

Equation numbers are placed flush with the right margin, as in the above numbered equations. Some situations require unique numbering. Please use the forms shown in the following examples when you encounter similar circumstances.

- (1) A set of equations of equal importance may be numbered to demonstrate that relationship, e.g., (1a), (1b), and (1c).
- (2) A principal equation and subordinate equations (those that define quantities or variables in that equation) may be numbered (1), (1a), (1b), etc.
- (3) If an equation is a variant of a previous equation (it may be separated from the original equation by other equations and/or by text), it may be numbered with the same number as the original and a prime, double prime, etc., as appropriate.

D. Bracketing

1. Grouping sequence

For the purpose of grouping, the sequence of bracketing preferred for *Reviews of Modern Physics* articles is $\{[(())]\}$, working outwards in sets $()$, $[\]$, and $\{\}$. If you have used these three sets and need additional bracketing, begin the sequence again in the same order but in bold print:

$$\{ [(\{ [()] \})] \}$$

For grouping situations that contain built-up material and need larger-sized bracketing, it is preferable to start again at the beginning of the sequence around the built-up material, e.g.,

$$\left[\left(\frac{(a-2)^{1/2}}{\alpha^2} \right) \left(\frac{(x+2)^{1/2}}{\beta} \right) \right] = 0. \quad (5)$$

2. Specific bracket notation

Bracketing (ordered and special) is also used to create specific notation that defines what it encloses. A list of approved specialized notation is included below. When used in an equation along with ordered bracketing, this special kind of bracketing should not alter the regular sequence of bracketing. The special notation $\langle \rangle$ in the following equation does not interfere with the sequence of the equation bracketing:

$$\hbar[\langle E - (a+1) \rangle]^{-1} = 0. \quad (6)$$

Please note the difference between $\langle \rangle$ (L^AT_EX: `\langle \rangle`, `\rangle`) and $\langle \rangle$ (L^AT_EX: `\rangle`, `\rangle`) the usual greater-than, less-than symbols.

3. Specialized bracket notation

Table VIII lists more uses of brackets, including crystallographic notations. These generally follow common usage and are included here for your convenience.

E. Additional style guidelines

1. Placement of limits and indices

In displayed math, limits and indices on sums, integrals, and similar symbols are handled in traditional ways:

$$\sum_{\substack{i,j,k \\ i < j < k}} \int_{-\infty}^{+\infty} \prod_{n > 1} \quad (7)$$

TABLE VIII Specialized bracket notation.

Object	Notation
Plane or set of parallel planes	(111)
Direction	[111]
Class (group) of symmetry equivalent directions	$\langle 111 \rangle$
Class (group) of symmetry equivalent planes	{111}
Point designated by coordinates	(x, y, z)
Lattice position in a unit cell (not bracketed)	$\frac{1}{2} \frac{1}{2} \frac{1}{2}$
Vector written in components	(H_x, H_y, \vec{H}_z)
Commutator	$[f, g]$
Anticommutator	$\{f, g\}$
Nested commutators	$[H_0, [H_0, H_1]]$
Functionals	$F[x]$
Sets	$\{x_i\}$
Absolute value, determinant	$ x , A $
Evaluation of a quantity	$ _{\phi_0=0}$
Norm	$\ A\ $
Average or expectation value	$\langle \rangle, \langle \rangle_{av}$

In text, however, space limitations require that single-limit sums or integrals use subscripts and superscripts, for example, $\sum_{n=1}^{\infty}$ and \int_0^a . Multiple-limit large symbols, such as the first sum in Eq. (7), should always be displayed.

2. Fractions

Fractions can be “built up” with a fraction bar, $\frac{a+b}{c}$, “slashed” with a solidus, $(a+b)/c$, or written with a negative exponent, $(a+b)c^{-1}$. In text all fractions must be either slashed or written with a negative exponent.

Observe the following guidelines on the use of fractions.

- (a) Use built-up fractions in matrices:

$$M_1 = - \begin{pmatrix} \frac{\partial^2}{\partial x^2} & 2\theta'_0 \frac{\partial}{\partial x} \\ \theta_0 \frac{\partial}{\partial x} & \theta_0^2 \frac{\partial^2}{\partial x^2} \end{pmatrix}. \quad (8)$$

- (b) Use built-up fractions in displayed equations:

$$H_A(w) = \left[\frac{1}{2} \left(\frac{Q}{\pi\omega^2} \right)^2 + \frac{c_e^2}{4d} \right] \pi\omega^2 d. \quad (9)$$

- (c) Using slashed fractions in subscripts, superscripts, limits, and indices is preferred:

$$N^{-1/2} \quad m_{3/2} \quad \int_{-\pi/2}^{\pi/2} \quad (10)$$

- (d) Use slashed or sized fractions in the numerators and denominators of built-up fractions except where excessive bracketing would obscure your meaning or slashing would interfere with continuity of notation:

$$\varphi + \frac{(\beta/6)\varphi}{\gamma + [\beta(\beta-1)/12]\varphi^2} = 0. \quad (11)$$

- (e) Be careful not to write ambiguous fractions when using the slashed notation; clearly indicate order of operations where necessary.

3. Multiplication signs

The primary use of the multiplication sign is to indicate a vector product of three-vectors (e.g., $\boldsymbol{\kappa} \times \mathbf{A}$). Do not use it to express a simple product except

- (1) when breaking a product from one line to another (described in Sec. XVI.C.3) or
- (2) for other cases such as indicating dimensions (e.g., $3 \times 3 \times 3 \text{ cm}^3$), magnification ($40\times$), symbols in figures (\times 's), or numbers expressed in scientific notation ($1.6 \times 10^{-19} \text{ C}$).

The center dot (\cdot) should not be used to mean a simple product. Use the dot to represent inner products of vectors ($\mathbf{k} \cdot \mathbf{r}$).

4. Mathematical terms

The use of the following standard symbols is recommended.

TABLE IX Standard symbols.

\sim	approximately or varies as
\simeq	approximately equal
\rightarrow	tends to
\propto	proportional to
$\mathcal{O}()$	of the order
A^*	complex conjugate of A
A^\dagger	Hermitian conjugate of A
A^T	transpose of A
\mathbf{k}	unit vector \mathbf{k}/k

5. Radical signs and overbars

You may use radical signs (roofed only, e.g., $\sqrt{\overline{xx}}$) and overbars (\overline{xx}) when they go over material of six or fewer characters that are without superscripts. If the material is longer or has superscripts, alternative notation should be used. For $\sqrt{\overline{xx}}$ use $(xx)^{1/2}$ and for \overline{xx} use $\langle xx \rangle$ or $\langle \overline{xx} \rangle_{av}$ if appropriate. If the overbar means complex conjugate, then $(xx)^*$ should be used. A radical sign (roofed) should not be used on built-up material, although an overbar can be used.