

# Reviews of Modern Physics Style Guide

## XVI. MATHEMATICAL MATERIAL

Mathematical symbols must be defined immediately where they are introduced. Exceptions are the fundamental constants: the velocity of light  $c$ , Planck's constant  $\hbar$ , the electronic charge  $e$ , Boltzmann's constant  $k$  or  $k_B$ , and the electron mass  $m_e$ .

Avoid using the same symbol for two different things. If you think that a list of symbols would be helpful to your readers, you can provide one preceding the references (see Sec. VI.).

In what follows, only the style of equations and symbols will be considered. Authors are urged to consult L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> and REV<sub>T</sub>E<sub>X</sub> 4-1 documentation for instructions on using these packages to display mathematics.

### A. Characters

#### 1. Character fonts

The italic font is used for mathematical symbols (this is the default font in L<sup>A</sup>T<sub>E</sub>X's math mode). In addition to variables and constants, the italic font is used for particle symbols, symbols of quantum states, and group-theoretic designations.

In general please use the following hierarchy of font styles for symbols:

TABLE I Font style for symbols.

Font style	Symbol	Example
lower case	variables, constants, and ordinary functions	$x, \alpha, f$
upper case	matrices and functions	S, F
script upper case	operators	$\mathcal{H}$
boldface lower case	three-vectors	$\mathbf{r}$
boldface upper case	matrices three-vectors	$\mathbf{J}, \mathbf{B}$

#### 2. Diacritical signs

A *diacritical sign* is a marking placed directly above or below symbols, e.g., the arrow in  $\vec{x}$ . It is possible to make multilevel marks – placing several diacritical marks above or below one letter or symbol – but this quickly becomes confusing. Restrict the number of such marks to two to avoid confusion. The underline can appear under any configuration.

For your convenience, Table II contains a list of commonly used diacritical marks along with the L<sup>A</sup>T<sub>E</sub>X code that produces them.

TABLE II Diacritical marks and associated L<sup>A</sup>T<sub>E</sub>X codes.

Diacritical mark	L <sup>A</sup> T <sub>E</sub> X code
$\vec{x}$	<code>\tensor{x}</code>
$\dot{x}$	<code>\dot{x}</code>
$\ddot{\mathbf{r}}$	<code>\ddot{\{\bf r}}</code>
$\hat{\theta}$	<code>\dot{\{\hat{\theta}}}</code>
$\underline{A}$	<code>\underline{A}</code>
$\overline{\overline{x + y}}$	<code>\overline{\overline{x + y}}</code>

#### 3. Subscripts and superscripts

All available characters can be used as subscripts or superscripts. Position of subscript or superscript is dictated by standard notation. In almost all cases you should set right and left subscripts and superscripts flush against the symbol they accompany (as in the following).

Examples:

$$R_0^x, {}^{110}\text{Ag}^m, \rho_0^{(N)}, \int_0^1, \sum', \lim_{l \rightarrow \infty} \text{ (in text)}$$

There are, however, some exceptions to this general rule. Examples appear below.

- tensor notation:  $g_{\mu\nu}(\phi^z)^{\alpha;\alpha}$
- molecular ions:  $\text{H}_2^+, \text{O}_2^-$
- footnotes in tables:  $E_n^a$

Presuperscripts or presubscripts are set flush against the symbols they accompany. In addition, it is advisable to insert an extra thin space between a presuperscript or presubscript and a preceding symbol in cases where clarity is questionable, i.e.,

$$8p\sigma^1\Sigma_u^+ \quad \text{or} \quad d^9s^z p^3P_2$$

The number of levels of subscripts and superscripts attached to a symbol will also affect clarity. Two double levels is generally considered the most complicated combination acceptable, i.e.,

$$M_{b_k^\dagger}^{a_i^2}$$

When additional indices are needed, insert a comma or thin space and keep the added indices on the same line, i.e.,

$$M_{b_k, d_p}, \quad \sigma_{r, s+1}, \quad \text{or} \quad \sigma_{r\ s+1}$$

## B. Abbreviations in math

Some abbreviations, such as those for mathematical functions and those used in superscripts or subscripts, require special handling and are discussed below.

### 1. Abbreviations designating mathematical functions

Multiletter abbreviations of mathematical functions are always written in the roman font (i.e.,  $\sin$ ). The standard trigonometric functions are  $\cos$ ,  $\cot$ ,  $\sec$ ,  $\sin$ , and  $\tan$ . Hyperbolic trigonometric functions add “h” to the end; the preferred notation for inverse functions is  $\tan^{-1}$  rather than  $\arctan$ . The preferred notation for the logarithm to the base  $e$  ( $\log_e$ ) is “ln”; “log” without a subscript denotes the logarithm to the base 10 ( $\log_{10}$ ).

The following guidelines are in general use in clear mathematical writing:

- (a) roman multiletter abbreviations must be closed up to the argument following and separated from any preceding symbol by a thin space, that is,

$$K \cos[Q(z - z_0)]$$

$$K \exp[x^2(b_2 + b_1)^{-\frac{1}{2}}]$$

- (b) In addition, by convention it is assumed that an argument ends as soon as another function appears, i.e.,  $\sin x \cos b$ , or at a plus or minus sign, i.e.,  $\sin x + y$ , but if other mathematics is involved or there is any ambiguity you should insert bracketing, as in the following examples:

$$\sin[-(x + a)], (\sin x)/a, \text{ and } \exp[x^2(b_z + b_1)^{-\frac{1}{2}}]$$

- (c) To treat a function of a function enclose it in bold round parentheses, i.e.,

$$g(f(x))$$

- (d)  $e$  and  $\exp$  (for exponent) notation follow both of the preceding conventions. Which form to use,  $e$  or  $\exp$ , is determined by the number of characters and the complexity of the argument. The  $e$  form is appropriate when the argument is short and simple, i.e.,  $e^{i\mathbf{k}\cdot\mathbf{r}}$ , whereas  $\exp$  should be used if the argument is more complicated.

### 2. Abbreviations in subscripts and superscripts

Abbreviations in subscripts and superscripts fall into two categories: (1) single letter and (2) multiletter. Most single-letter abbreviations are conventionally printed in the italic font, i.e.,  $E_C$  where  $C$  stands for Coulomb. Multiletter abbreviations are conventionally printed in the roman font whether they represent one or more words,

i.e.,  $E_{\text{lab}}$ , where lab stands for laboratory and  $E_{\text{HF}}$ , where HF stands for Hartree and Fock, two proper names. Please note that you should always capitalize abbreviations that represent proper names.

When you are creating your own abbreviations, do not put periods in acronyms (whether in line or in subscripts), but do insert them if you are abbreviating words that are headings in a table.

## C. Mathematical expressions

### 1. When to display

Mathematical material that is set apart from the main text in the traditional manner is referred to as *displayed* material. In general, authors should observe the following guidelines: Display (1) equations of importance, (2) all equations that are numbered, (3) those that are too long to fit easily in text (over 25 characters), or (4) those that are complicated (contain built-up fractions, matrices, or matrixlike expressions). Consider, also, displaying math that contains multilevel indices, integral, summation, and product signs, with multilevel or complex limits, or any other situation in a formula that creates the need for extra vertical spacing in a text line.

### 2. Punctuation

Even though displayed math is separated by space from the running text it still is a part of that text and needs to be punctuated accordingly. The following is an example.

The final result is

$$H_{ij} = \left(\frac{\Omega}{\Delta}\right)^2 \frac{|J|^2}{E_g + \frac{1}{2}(W_c + W_v)} e^{\lambda\mathbf{K}\cdot\mathbf{R}_{ij}}, \quad (1)$$

where

$$\mathbf{K} = \frac{1}{a}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \quad (2)$$

and

$$\lambda = \ln [W_l W_v / (12E_g)^2]. \quad (3)$$

Note the use of the comma at the end of the first equation, and the period at the end of the third equation.

### 3. Equation breaking (multiline equations)

Mathematical expressions often need to be displayed on two or more lines (“broken”) because of the line length limitations of the *Reviews of Modern Physics* standard



TABLE III Specialized bracket notation.

Object	Notation
Plane or set of parallel planes	(111)
Direction	[111]
Class (group) of symmetry equivalent directions	$\langle 111 \rangle$
Class (group) of symmetry equivalent planes	{111}
Point designated by coordinates	$(x, y, z)$
Lattice position in a unit cell (not bracketed)	$\frac{1}{2} \frac{1}{2} \frac{1}{2}$
Vector written in components	$(H_x, H_y, \vec{H}_z)$
Commutator	$[f, g]$
Anticommutator	$\{f, g\}$
Nested commutators	$[H_0, [H_0, H_1]]$
Functionals	$F[x]$
Sets	$\{x_i\}$
Absolute value, determinant	$ x ,  A $
Evaluation of a quantity	$ _{\phi_0=0}$
Norm	$\ A\ $
Average or expectation value	$\langle \rangle, \langle \rangle_{av}$

In text, however, space limitations require that single-limit sums or integrals use subscripts and superscripts, for example,  $\sum_{n=1}^{\infty}$  and  $\int_0^a$ . Multiple-limit large symbols, such as the first sum in Eq. (7), should always be displayed.

## 2. Fractions

Fractions can be “built up” with a fraction bar,  $\frac{a+b}{c}$ , “slashed” with a solidus,  $(a+b)/c$ , or written with a negative exponent,  $(a+b)c^{-1}$ . In text all fractions must be either slashed or written with a negative exponent.

Observe the following guidelines on the use of fractions.

- (a) Use built-up fractions in matrices:

$$M_1 = - \begin{pmatrix} \frac{\partial^2}{\partial x^2} & 2\theta'_0 \frac{\partial}{\partial x} \\ \theta_0 \frac{\partial}{\partial x} & \theta_0^2 \frac{\partial^2}{\partial x^2} \end{pmatrix}. \quad (8)$$

- (b) Use built-up fractions in displayed equations:

$$H_A(w) = \left[ \frac{1}{2} \left( \frac{Q}{\pi\omega^2} \right)^2 + \frac{c_e^2}{4d} \right] \pi\omega^2 d. \quad (9)$$

- (c) Using slashed fractions in subscripts, superscripts, limits, and indices is preferred:

$$N^{-1/2} \quad m_{3/2} \quad \int_{-\pi/2}^{\pi/2} \quad (10)$$

- (d) Use slashed or sized fractions in the numerators and denominators of built-up fractions except where excessive bracketing would obscure your meaning or slashing would interfere with continuity of notation:

$$\varphi + \frac{(\beta/6)\varphi}{\gamma + [\beta(\beta-1)/12]\varphi^2} = 0. \quad (11)$$

- (e) Be careful not to write ambiguous fractions when using the slashed notation; clearly indicate order of operations where necessary.

## 3. Multiplication signs

The primary use of the multiplication sign is to indicate a vector product of three-vectors (e.g.,  $\boldsymbol{\kappa} \times \mathbf{A}$ ). Do not use it to express a simple product except

- (1) when breaking a product from one line to another (described in Sec. .C.3) or
- (2) for other cases such as indicating dimensions (e.g.,  $3 \times 3 \times 3 \text{ cm}^3$ ), magnification ( $40\times$ ), symbols in figures ( $\times$ 's), or numbers expressed in scientific notation ( $1.6 \times 10^{-19} \text{ C}$ ).

The center dot ( $\cdot$ ) should not be used to mean a simple product. Use the dot to represent inner products of vectors ( $\mathbf{k} \cdot \mathbf{r}$ ).

## 4. Mathematical terms

The use of the following standard symbols is recommended.

TABLE IV Standard symbols.

$\sim$	approximately or varies as
$\simeq$	approximately equal
$\rightarrow$	tends to
$\propto$	proportional to
$\mathcal{O}()$	of the order
$A^*$	complex conjugate of $A$
$A^\dagger$	Hermitian conjugate of $A$
$A^T$	transpose of $A$
$\hat{\mathbf{k}}$	unit vector $\mathbf{k}/k$

## 5. Radical signs and overbars

You may use radical signs (roofed only, e.g.,  $\sqrt{\overline{xx}}$ ) and overbars ( $\overline{xx}$ ) when they go over material of six or fewer characters that are without superscripts. If the material is longer or has superscripts, alternative notation should be used. For  $\sqrt{\overline{xx}}$  use  $(xx)^{1/2}$  and for  $\overline{xx}$  use  $\langle xx \rangle$  or  $\langle xx \rangle_{av}$  if appropriate. If the overbar means complex conjugate, then  $(xx)^*$  should be used. A radical sign (roofed) should not be used on built-up material, although an overbar can be used.